COMPUTATIONAL METHODS AND IMPLEMENTATION TECHNIQUES FOR PROBLEMS RELATED TO NAVIGATION, GUIDANCE, AND CONTROL

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A Descent Algorithm for Constrained Stochastic Extremum - Y.C. Ho & P.M. Newbold

Consider the problem of determining $u(t)$, $t_o \leq t \leq t_f$ to minimize

$$J = E\left[\phi\left[x(t_f)\right] + \int_{t_o}^{t_f} L(x,u,t)dt\right]$$

subject to the constraint

$$\dot{x} = f(x,u,t) + g(x,t)w \quad , \quad x(t_o) \text{ specified}$$

where $w(t)$ is a white random process with known statistical characteristics. Such problems are called open-loop stochastic control problems [1]. A decent scheme for determining the extremal $u(t)$ has been devised which is the direct analog of the deterministic gradient method. The convergence of the algorithm with probability one has been proved using the results from the stochastic stability theory. Two sample problems have been simulated: (i) optimal ascent trajectory with uncertain thrust, and (ii) optimal filter for system with uncertain dynamics. The results support the theory developed. A technical report has now been published (Cruft Lab. Report No. 524).

Effect of Word Length Error in the Mechanization of Navigation Experiments - J.S. Lee

Since the word length of an on-board computer is usually limited, the round-off error introduced is another source of random error in navigation computation. A model which governs the propagation of these errors as a function or word length used has been developed. The theoretically predicted results agree well with some experimental data obtained at NASA-ERC [2].

The Conjugate Gradient Method Applied to a Maximum Range Aircraft Problem - M. Desai

The conjugate gradient method consistently worked better than steepest descent methods, when applied to parameter optimization problems. Problems with equality constraints were handled satisfactorily by using penalty functions.

The conjugate gradient method also worked well on simple optimal control problems involving nonlinearity and terminal constraints. In particular, it worked very well on a problem often used as a test of new methods in the literature, namely low-thrust orbital transfer from Earth's orbit to Mar's orbit.

Currently, the conjugate gradient method is being applied to a difficult optimal control problem, the maximization of horizontal range of a jet aircraft for a given amount of fuel.

Smoothing for Linear Continuous Dynamic Systems When Measurements Contain Correlated Noise - R.K. Mehra and A. E. Bryson, Jr.

This paper, which will soon be published as a Harvard Cruft Technical Report, treats the problem as a calculus of variations problem with equality constraints. The filtering results of Bryson and Johansen follow as part of the solution to the smoothing problem [3].

Flight Path Optimization of a V/STOL Aircraft - R.K. Mehra & A. E. Bryson, Jr.

Optimization of flight-paths for V/STOL aircraft using the "conjugate gradient method" is a program that was written for the CTSS (Compatible Time-Sharing System, at Massachusetts Institute of Technology) which solves general calculus of variations problems and is being used to obtain optimum flight paths for V/STOL aircraft.

The Separate Computation of Arcs for Optimal Flight Paths with State Variable Inequality Constraints - J.L. Speyer, R.K. Mehra & A. E. Bryson, Jr.

Separate computation of arcs is possible for a large class of optimization problems with state variable inequality constraints. Surprisingly, this class (to the best of the authors' knowledge) includes all physical problems which have been solved analytically or numerically to date. Typically these problems have only one constrained arc. Even in more complex problems, separation of arcs can be used to search for additional constrained arcs.

As an important example, a maximum range trajectory for a glider entering the Earth's atmosphere at a supercircular velocity is determined subject to a maximum altitude constraint after initial pull-up. It is shown that the optimal path can be divided into three arcs, which may be determined separately with no approximations. The three arcs are: (1) the initial arc, beginning at specified initial condition and ending at the entry point onto the altitude constraint; (2) the arc lying on the altitude constraint; and (3) the terminal arc, beginning at the exit point of the altitude constraint and ending at some specified terminal altitude.

The conjugate gradient methods, a first order optimization scheme, is shown to converge very rapidly to the individual unconstrained optimal arcs. Using this optimization and taking advantage of the separation of arcs, an investigation revealed that two locally optimum paths exist. The range of one exceeds the range of the other by about 250 nautical miles (about 6%) for the re-entry vehicle used here (maximum lift-to-drag ratio is .9).

A Neighboring Optimum Feedback Control Scheme Based on Estimated Time-to-go With Application to Re-entry Flight Paths. J.L. Speyer & A. E. Bryson, Jr.

A modification of the perturbation feedback control scheme of [4], [5], and [6] is presented that greatly increases its capability to handle disturbances in cases where the final time is not specified. The modified control scheme uses a set of precalculated gains which allows in-flight estimation of the change in the final time due to perturbations from a nominal path. The time-to-go, determined from the predicted change in final time, is used to enter cables of precalculated feedback control gains.

This modified guidance scheme is applied to a re-entry glider entering the atmosphere of the Earth at supercircular velocities. Beginning at the bottom of the pull-up maneuver, the glider is guided to a terminal altitude of 220,000 feet and zero (0) flight path angle with maximum terminal velocity. For initial altitudes between 167,000 and 216,000 feet the terminal error in altitude is less than two feet; for initial velocities between 23,000 ft./sec. and 43,000 ft./sec. the terminal altitude error is less than 13 feet. In addition the terminal velocity is very close to optimal for these initial conditions.

The suggestion for using such a scheme was first given by Kelley [7] who used performance index-to-go as the index variable. He called this a "transversal comparison" scheme. Using time-to-go for the transversal comparison is more general since time-to-go is always monotonically decreasing whereas this is not always true of performance index-to-go. Clearly a monotonically changing index variable must be used since the transversal comparison should be made over the entire flight. By an iterative scheme the transversal comparison is used in predicting the time-to-go. Kelley's suggestion is also extended here to include non-stationary systems and changes in the terminal constraints.

Estimation Using Sampled-Data Containing Sequentially Correlated Noise - L. J. Henrikson and A. E. Bryson, Jr.

The problem considered is that of estimating the state variables of a multi-stage, linear dynamic system based on measurements of linear combinations of the state variables containing additive sequentially correlated noise. A design procedure for the data processing estimation filters is developed which eliminates the ill-conditioned computations of the augmented state approach, and which is of a lower dimension than the augmented state filters.

The present design procedure was suggested by the work of Bryson and Johansen [3] on the related problem for continuous linear dynamic systems. Considering the measurement vector as a set of constraints among the augmented state variables, a measurement differencing scheme is used to reduce the dimension of the estimation problem. The estimation theory of Kalman [8] is then applied to this reduced problem. This work will be presented at the AIAA Guidance, Control, and Flight Dynamics Conference, Huntsville, Alabama, August 14-16, 1967.

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